

# Contents

<b>Notations</b>	<b>9</b>
<b>1. Introduction</b>	<b>13</b>
<b>2. Literature review</b>	<b>19</b>
2.1. The assumed stress fields in linear theory of elasticity . . . .	20
2.2. Linear plate theories . . . . .	23
2.3. Finite element methods for plate bending . . . . .	26
2.3.1. Displacement method . . . . .	27
2.3.2. Equilibrium models . . . . .	32
2.3.3. Mixed and hybrid methods . . . . .	35
2.3.4. Other methods and techniques in plate analysis . . . .	40
2.4. Duality in finite element methods . . . . .	42
<b>3. Plate bending by equilibrium finite element method</b>	<b>45</b>
3.1. Introduction . . . . .	45
3.2. Problem formulation . . . . .	46
3.2.1. Governing equations for an elastic plate . . . . .	46
3.2.2. Boundary conditions . . . . .	47
3.3. Complementary work principle . . . . .	48
3.4. Construction of statically admissible stress fields . . . . .	51
3.5. Matrix form of the equilibrium model . . . . .	52
3.6. Boundary conditions . . . . .	55
3.6.1. Simply supported edge . . . . .	56
3.6.2. Free edge . . . . .	57
3.6.3. Clamped edge . . . . .	61
3.7. Applied elements . . . . .	61
3.7.1. Triangular plate bending elements . . . . .	61
3.7.2. Multi-point constraints elements . . . . .	72

<b>4. Displacement field by the least squares method</b>	<b>79</b>
4.1. Introduction . . . . .	79
4.2. Formulation of the problem . . . . .	80
4.3. The discretized variational problem . . . . .	82
4.4. Applied element . . . . .	84
4.5. Least squares method in matrix form . . . . .	86
<b>5. Error estimation of approximate solution - Prager–Synge’s method</b>	<b>91</b>
5.1. Definition of statically and kinematically admissible solutions	93
5.2. Dual variational principles . . . . .	94
5.3. Upper and lower bounds for the relative error . . . . .	97
5.4. Additional remarks . . . . .	100
<b>6. Numerical examples</b>	<b>101</b>
6.1. Displacement-based plate elements used for comparison . . .	102
6.2. MWLS method . . . . .	103
6.3. Circular clamped plate . . . . .	104
6.4. Square plate with two simply-supported and two clamped edges	112
6.5. Trapezoidal plate with point-supports . . . . .	117
6.6. Rectangular orthotropic plate . . . . .	125
6.7. Plates subjected to concentrated loads . . . . .	131
6.8. Trapezoidal plate with one free edge . . . . .	135
<b>7. Conclusions</b>	<b>141</b>
<b>List of figures</b>	<b>145</b>
<b>List of tables</b>	<b>147</b>
<b>Abstract</b>	<b>151</b>
<b>Streszczenie</b>	<b>155</b>
<b>Zusammenfassung</b>	<b>159</b>
<b>Bibliography</b>	<b>163</b>
<b>A. Beam bending by equilibrium finite element method</b>	<b>181</b>
A.1. Introduction . . . . .	181
A.2. Problem formulation . . . . .	182

---

A.3. Variational form of the problem: the complementary work principle . . . . .	183
A.4. Matrix form of the equilibrium model . . . . .	184
A.5. Boundary conditions . . . . .	186
A.6. Applied element . . . . .	188
A.7. Numerical examples . . . . .	190